Surface Impedance Characterization for UTD Based Solution with IBC for Surface Fields on a Dielectric-Coated PEC Circular Cylinder

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***Abstract*—A novel formulation for the surface impedance characterization is introduced for the canonical problem of surface fields on a perfect electric conductor (PEC) circular cylinder with a dielectric coating due to a electric current source using the Uniform Theory of Diffraction (UTD) with an Impedance Boundary Condition (IBC). The approach is based on a TE/TM assumption of the surface fields from the original problem. Where this surface impedance fails, an optimization is performed to minimize the error in the SD Green's function between the original problem and the equivalent one with the IBC. This new approach requires small changes in the available UTD based solution with IBC to include the geodesic ray angle and length dependence in the surface impedance formulas. This asymptotic method, accurate for large separations between source and observer points, in combination with spectral domain (SD) Green's functions for multidielectric coatings leads to a new hybrid SD-UTD with IBC to calculate mutual coupling among microstrip patches on a multilayer dielectric-coated PEC circular cylinder. Results are compared with the eigenfunction solution in SD, where a very good agreement is met.**

Keywords - **Conformal array antennas, hybrid electromagnetic methods, Green's function, uniform theory of diffraction (UTD), surface impedance.**

I. Introduction

New communications and radar systems have demanding requirements which lead to the research on novel antenna configurations. Conformal array antennas may be an alternative to planar antennas because they can be integrated into a curved surface offering large observation angles, low aerodynamic payload and aesthetic advantages, which make them very attractive for satellites, aircrafts, ships, land vehicles or ground base stations. Microstrip antennas on multilayer dielectric structures are extensively used due to their low fabrication cost, light weight, ease of conformity on curved surfaces, and direct integrability with active devices.

For electrically small or medium size conformal array antennas, modal solutions in spectral domain (SD) are very convenient for canonical convex shapes such as circular cylinder [1], but as antenna dimensions increase a large number of terms are necessary in the integral equations. Moreover, with the SD approach dielectric-coated antennas with several layers can be analyzed efficiently, like in G1DMULT algorithm [2]. For electrically large structures the uniform theory of diffraction (UTD) based asymptotic method [3] is commonly used to calculate surface fields for canonical and arbitrarily convex shaped PEC surfaces. On the other hand, it cannot support multilayer dielectric coatings and requires a large separation between source and observation points.

Recently, a new UTD based asymptotic solution with impedance boundary conditions (IBC) has been introduced for surface field determination on a circular cylinder [4]. Current solution derives Green's functions transforming the original problem, a dielectric-coated PEC circular cylinder, into a equivalent problem, a circular cylindrical surface impedance, by setting an IBC. It performs an asymptotic expansion of surface fields through Watson transformation and the steepest descent path (SDP) method to get spatial domain asymptotic Green's functions. UTD with IBC Green's functions combined with Method of Moments (MoM) are able to obtain the mutual coupling between patches. Nevertheless, although some progress has been done to calculate efficiently proper Green's functions [5], very few attention has been paid about how to figure out surface impedance for each specific case, because it must change with the inclination angle of the geodesic rays along the structure.

In this paper, the surface impedance is derived in the SD by performing a TE/TM decomposition of the surface fields. UTD based asymptotic Green's functions with IBC are slightly modified to include the surface impedance dependence with the geometrical parameters of the rays upon the cylinder. Since UTD with IBC solution would be valid only for one dielectric layer and for a large separation between the source and the observation point, it is proposed its hybridization with the SD approach, thus extending the method to multilayer structures and increasing the accuracy of the surface fields calculated. In this work, nonparaxial region has only been considered.

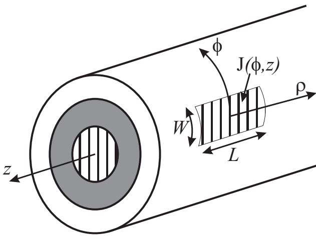


Fig. 1. Patch on a multilayer dielectric-coated PEC circular cylinder.

II. Method of Analysis

A. Hybrid SD-UTD with IBC

The geometry of the problem is a perfectly conducting patch located over the surface of a multilayer dielectric-coated PEC circular cylinder, as is shown in Fig. 1. It is assumed that the cylinder is infinite in the axial direction, thus reducing the three-dimensional problem into a one-dimensional one if fields and currents are expressed in the SD.

The Electric Field Integral Equation (EFIE) formulation starts by enforcing the boundary condition where the total tangential electric field must be zero on the surface of the patch. The patch current is calculated using MoM, and the impedance matrix elements are

$$Z_{ji} = \frac{1}{4\pi^2} \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{\tilde{W}}_j(-m, -k_z) \cdot \underline{\underline{\tilde{G}}}^{e,J}(m, k_z) \cdot \underline{\tilde{J}}_i(m, k_z) dk_z \quad (1)$$

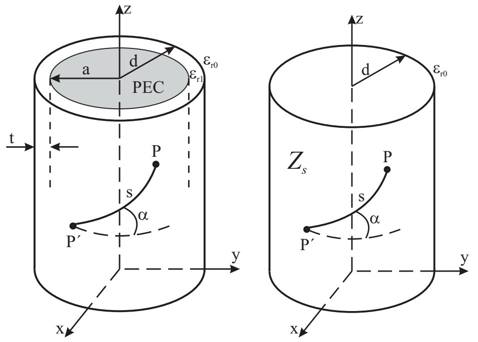
where tilde $\tilde{X}$ indicates that function is in SD. The $\underline{\underline{\tilde{G}}}^{e,J}$ is the SD Green's function for the surface electric field due to an electric current source on the multilayer dielectric-coated PEC circular cylindrical structure, which is calculated by using the numerical algorithm G1DMULT [2].

This approach can be accelerated combining an asymptotic UTD based solution of the surface fields, as it was proposed in [6] for aperture conformal array antennas. The hybrid method consists into a extraction procedure of the Green's function in the MoM matrix element, as

$$\begin{aligned} Z_{ji} = \frac{1}{4\pi^2} &\sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{\tilde{W}}_j(-m, -k_z) \\ &\cdot (\underline{\underline{\tilde{G}}}^{e,J}(m, k_z) - \underline{\underline{\tilde{G}}}^{e,J}_{asym}(m, k_z) -) \cdot \underline{\tilde{J}}_i(m, k_z) dk_z \\ &+ \int_S \int_{S'} \underline{W}_j(\phi, z) \cdot \underline{\underline{G}}^{e,J}_{asym}(d, \phi - \phi', z - z') \\ &\cdot \underline{J}_i(\phi', z') dS dS' \end{aligned} \quad (2)$$

where $\underline{\underline{G}}^{e,J}_{asym}$ is the spatial domain asymptotic UTD based Green's function.

To simplify the computation of this asymptotic part, the boundary conditions on a dielectric coated PEC surface can be approximated by an IBC



(a) Original problem. (b) Equivalent problem.

Fig. 2. Geometry of a circular cylinder.

$$\left[\begin{array}{c} \tilde{E}_z(\underline{r}) \\ \tilde{H}_z(\underline{r}) \end{array} \right] = \left[\begin{array}{cc} Z_{sm} & 0 \\ 0 & -Z_{se}^{-1} \end{array} \right] \left[\begin{array}{c} \tilde{E}_\phi(\underline{r}) \\ \tilde{H}_\phi(\underline{r}) \end{array} \right] \Bigg|_{\rho=d} \quad (3)$$

Two different surface impedance are used in (3), whose derivation is shown in section II-B. Thus, the problem of calculating Green's functions due to an electric point source on a dielectric coated PEC circular cylinder, Fig. 2(a), is reduced to an equivalent problem of a circular cylinder with a surface impedance Z_s, Fig. 2(b), where Green's functions are extracted from the IBC and asymptotically treated.

Asymptotic Green's functions for the electric field due to an electric source are the dual from those appearing in [4] but including a τ dependence on the surface impedance. Essentially, the Watson transformation is applied to the vector potentials calculated from IBC. Then, integral over spectral variable k_z can be performed through SDP procedure, assuming a separation s between source and field points large compared with the wavelength. Fock substitution ($\nu = k_{\rho 0} d + m_t \tau$) is applied to replace integration in the ν-plane by an integration in the τ-plane, yielding to the so-called Fock-type integrals. In the final field expressions, higher orders derivatives of Fock-type integrals are kept to increase accuracy in the asymptotic method, and they include the two surface impedances $Z_{se,m}$. Fock-type integrals can be solved efficiently through a numerical integration by deforming the integration path, as was shown in [5], which can be seen in Fig. 3, and using the Gauss-Kronrod quadrature. For all examples showed in section III, parameters in the integration path of all Fock-type integrals used have been set to: $\varepsilon = 0.001 k_0 d$, $\tau_a = -1 k_0 d$ and $\tau_b = 3 k_0 d$, with k_0 the upper layer wavenumber.

B. Surface Impedance Characterization

In scattering problems where vertically (TE) or horizontally (TM) polarized planar waves impinge over a certain slab, surface impedance can be modeled depending on the polarization [7]. In this document, a similar procedure is followed. This method consists into assume a TE/TM decomposition of the fields propagating along the cylinder. The IBC in (3) suggests that this approximation is acceptable and a different surface impedance is used depending on the electric and magnetic

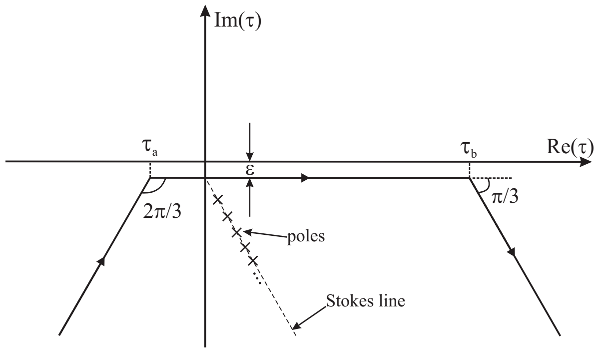


Fig. 3. Deformed integration path and poles location in the complex τ-plane for the Fock-type integrals computation.

field ratios. TE/TM surface impedance, from the eigenfunction solution of Fig. 2(a), is expressed in terms of Bessel functions of first and second kind as

$$Z_{se}(m,k_{\rho 1}) = -\left.\frac{\tilde{E}_\phi(\underline{r})}{\tilde{H}_z(\underline{r})}\right|_{\rho=d} =$$
$$-\frac{jk_0 Z_{c0}}{k_{\rho 1}} \cdot \frac{J'_m(k_{\rho 1}d)Y'_m(k_{\rho 1}a) - J'_m(k_{\rho 1}a)Y'_m(k_{\rho 1}d)}{J_m(k_{\rho 1}d)Y'_m(k_{\rho 1}a) - J'_m(k_{\rho 1}a)Y_m(k_{\rho 1}d)} \quad (4)$$

$$Z_{sm}(m,k_{\rho 1}) = \left.\frac{\tilde{E}_z(\underline{r})}{\tilde{H}_\phi(\underline{r})}\right|_{\rho=d} =$$
$$\frac{jk_{\rho 1} Z_{c0}}{\varepsilon_{r1} k_0} \cdot \frac{J_m(k_{\rho 1}d)Y_m(k_{\rho 1}a) - J_m(k_{\rho 1}a)Y_m(k_{\rho 1}d)}{J'_m(k_{\rho 1}d)Y_m(k_{\rho 1}a) - J_m(k_{\rho 1}a)Y'_m(k_{\rho 1}d)} \quad (5)$$

where Z_{c0} and k_0 are the characteristic impedance and the wavenumber of the upper dielectric slab, respectively; $k_{\rho 1}$ is the spectral variable in the ρ-direction of the lower interface, defined as $k_{\rho 1}^2 = k_1^2 - k_z^2$, with k_1 the wavenumber of the lower dielectric and with m the spectral variable in the ϕ-direction. This TE/TM surface impedance is shown to be valid for low α angles and only for for zz' and $\phi\phi'$ field and source orientation.

It is possible to make an optimization of the TE/TM surface impedance for the cases where they fail by considering TE surface impedance as

$$Z_{se}^{opt}(m,k_{\rho 1}) = K \cdot Z_{se}(m - |m_{ibc} - m_{eigen}|, k_{\rho 1}) \quad (6)$$

The shifting factor $|m_{ibc} - m_{eigen}|$ is determined by finding the zero of the proper SD Green's function of the original problem m_{eigen} [8] (Fig. 2(a)) and for the equivalent problem with the IBC m_{ibc} [4] (Fig. 2(b)). Thus, the root of Z_{se}, where the Green's function of the original problem has the zero, is shifted. The proportionality factor K is calculated by optimizing its value through Simplex method, where the goal function is to minimize the average relative error between the amplitudes of the SD Green's function for the original problem and for the equivalent problem within a certain range of values m_k, i.e.

$$F_{error} = \frac{1}{M}\sum_{k=1}^{M} \frac{||\tilde{G}_{uv'}^{eigen}(m_k,k_z)| - |\tilde{G}_{uv'}^{ibc}(m_k,k_z)||}{|\tilde{G}_{uv'}^{eigen}(m_k,k_z)|} \quad (7)$$

Surface impedance equations (4) and (5) involve products of Bessel functions where numerical problems appear for large orders. Therefore, to avoid this numerical instabilities and increase computation speed two-term Debye's asymptotic formulas have been implemented, and Olver's asymptotic representation is used where they fail [9].

Furthermore, to increase the computation efficiency in the surface impedance formulas when calculating Fock-type integrals, they can be approximated with a ratio of two nth-order polynomials as a function of the spectral variable $k_t = m/d$ [10], as follows

$$Z_{se,m}(\tau) \sim \frac{a_n k_t^n + \ldots + a_2 k_t^2 + a_1 k_t + a_0}{b_n k_t^n + \ldots + b_2 k_t^2 + b_1 k_t + b_0} \quad (8)$$

Because k_t depends on τ, after Fock substitution, is a complex number and is written as

$$k_t(\tau) = k_0 \cos\alpha + \frac{m_t}{d}\tau \quad (9)$$

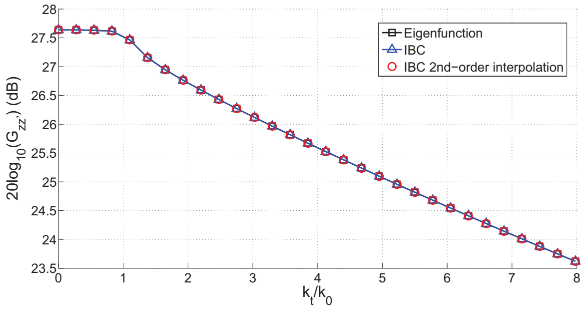
where m_t is a geometrical parameter.

In general, between a second-order and fourth-order polynomial is enough to fit surface impedance with a good precision, noting that a very high order could derive in a oscillatory behavior and in numerical difficulties.

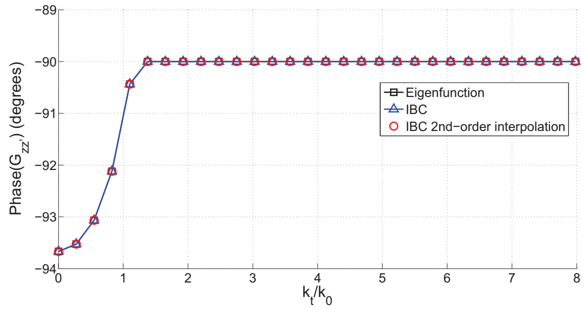
III. Results

To validate the TE/TM surface impedance hypothesis some results have been performed. The geometry considered is a large radius cylinder for a frequency of $4GHz$ with $a = 3\lambda_0$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r0} = 1$ and $t = 0.762mm$. For this dimensions, Fig. 4-6 show a comparison of the SD Green's functions in amplitude and phase, versus k_t, between the eigenfunction solution and the IBC with and without polynomial interpolation, for different source and field orientations and α angles. Surface impedance optimization is needed for angles larger than 25º for zz' and $\phi\phi'$ cases and for all α angles for $z\phi'$ component. By reciprocity, $\phi z'$ SD Green's function has the same analytical result than $z\phi'$. A very good agreement is met in SD, even for $z\phi'$ where non-optimized TE/TM surface impedance approach is not valid because couplings between modes are very strong. Although, in $z\phi'$ there is some difference for low k_t values, it must not affect final spatial domain surface field results because they are expanded asymptotically.

To access the accuracy of asymptotic representation of the surface fields some numerical results have been carried out for the mutual impedance, taking the second term of equation (2), between two tangential electric current modes located over the surface of a grounded dielectric-coated cylinder. The selected geometry is a cylinder with $\varepsilon_{r1} = 3.25$, $\varepsilon_{r0} = 1$ and $t = 0.06\lambda_0$ and with small dipoles with a width of $W = 0.02\lambda_0$



(a) Amplitude.



(b) Phase.

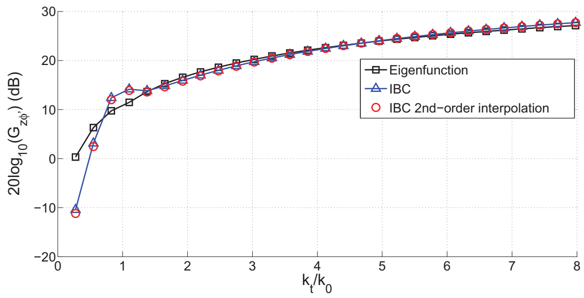
Fig. 4. SD Green's function for zz' component, with $f = 4GHz$, $a = 3\lambda_0$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r0} = 1$, $t = 0.762mm$ and $\alpha = 0^o$.

and a length of $L = 0.05\lambda_0$ in the direction of the current. Electric current modes are defined by sinusoidal along the direction of the current and by constant along the directions perpendicular to the current. In Fig. 7 and 8 are seen the mutual impedance for zz' direction, whose values are comparable with results in reference [11], where eigenfunction solution along with other asymptotic solution is shown. For these geometries, surface impedance optimization is not necessary, and a fourth order polynomial interpolation has been used.

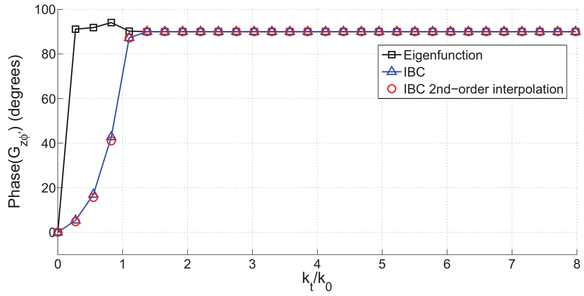
IV. Conclusion

A novel surface impedance approach has been introduced based on a TE/TM decomposition of the surface fields, to include geodesic ray orientation in a UTD based solution with IBC for surface fields on source excited dielectric-coated PEC circular cylinder. This method in combination with an eigenfunction algorithm in SD for multilayer coatings, can lead to an efficient hybrid SD-UTD with IBC, where MoM is used to solve an EFIE.

TE/TM surface impedance is accurate for low ray angles over the cylinder and only for zz' and $z\phi'$ source and field orientations. For other cases an optimized surface impedance is used by shifting the root position of the TE surface impedance and by tracking its slope to minimize the relative mean error between the eigenfunction and the IBC Green's functions. To improve the computation speed of the surface impedance a polynomial interpolation has been used, where between a second and a fourth order is necessary to fit curves accurately.

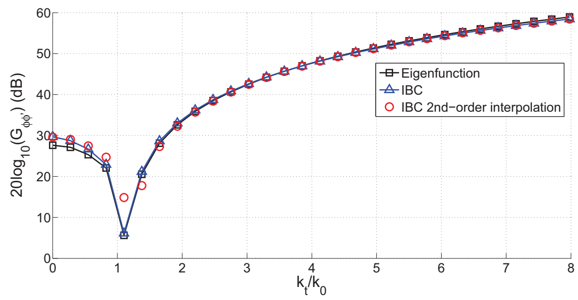


(a) Amplitude.

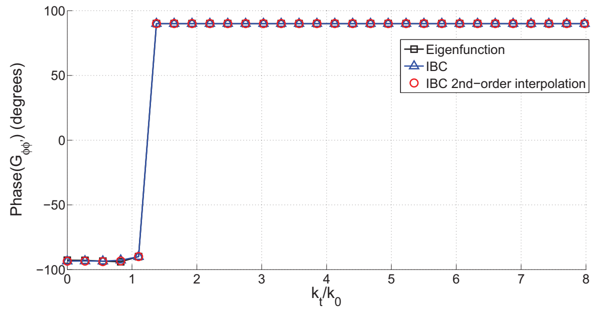


(b) Phase.

Fig. 5. SD Green's function for $z\phi'$ component, with $f = 4GHz$, $a = 3\lambda_0$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r0} = 1$, $t = 0.762mm$ and $\alpha = 60^o$.



(a) Amplitude.



(b) Phase.

Fig. 6. SD Green's function for $\phi\phi'$ component, with $f = 4GHz$, $a = 3\lambda_0$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r0} = 1$, $t = 0.762mm$ and $\alpha = 40^o$.

Results in SD are very promising showing a good agreement between eigenfunction solutions and the proposed surface impedance method. Even the optimized surface impedance works properly for the examples shown. Such a SD Green's

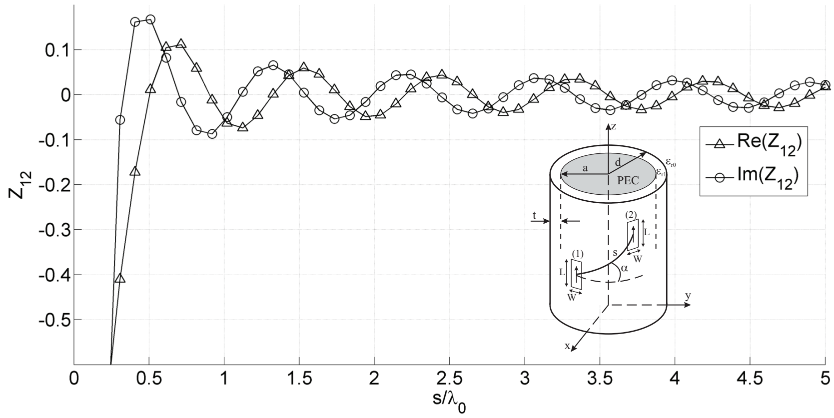


Fig. 7. Mutual impedance for zz' component, with $a = 3\lambda_0$, $\varepsilon_{r1} = 3.25$, $\varepsilon_{r0} = 1$, $t = 0.06\lambda_0$, $W = 0.02\lambda_0$, $L = 0.05\lambda_0$ and $\alpha = 55^{\text{o}}$.

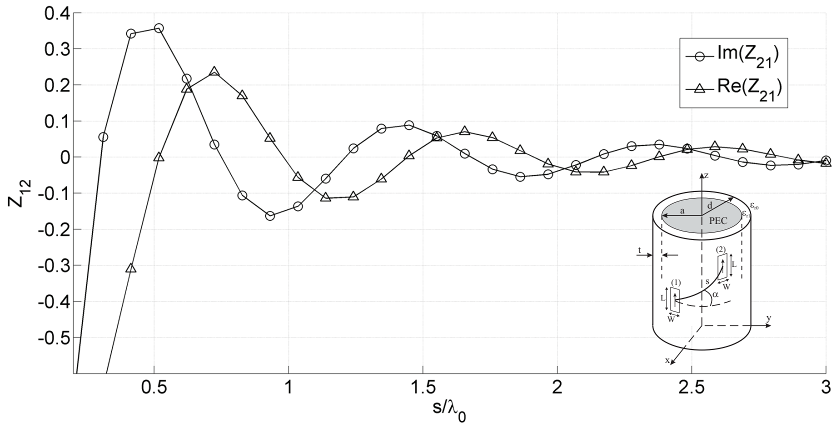


Fig. 8. Mutual impedance for zz' component, with $a = 1.5\lambda_0$, $\varepsilon_{r1} = 3.25$, $\varepsilon_{r0} = 1$, $t = 0.06\lambda_0$, $W = 0.02\lambda_0$, $L = 0.05\lambda_0$ and $\alpha = 40^{\text{o}}$.

functions are very stable avoiding numerical instabilities when computing Fock-type integrals. Some small differences are expected between eigenfunction solution and the UTD with IBC based procedure which is compensated in the hybrid method. Mutual coupling results with the hybrid SD-UTD with IBC will be presented at EuCAP 2012 meeting.

Acknowledgment

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